FP1 Specimen (IAL) (MA)

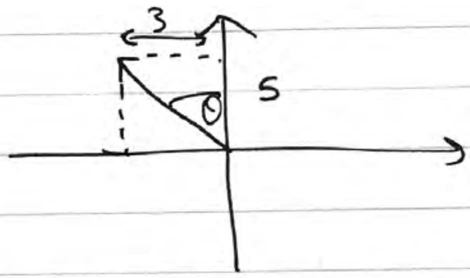
Q1a) $\dfrac{z_1}{z_2} = \dfrac{2+8i}{1-i} = \dfrac{(2+8i)(1+i)}{(1-i)(1+i)}$

$= \dfrac{2+2i+8i+8i^2}{1+\cancel{i}-\cancel{i}-i^2} = \dfrac{-6+10i}{2}$

$= \boxed{-3+5i}$

b) $\left|\dfrac{z_1}{z_2}\right| = \sqrt{3^2+5^2} = \boxed{\sqrt{34}}$

c) $\tan\theta = \dfrac{3}{5}$



$\theta = \tan^{-1}\dfrac{3}{5} = 30.96°..$

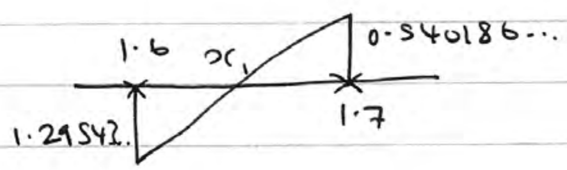
$= 0.5404^c..$

$\therefore \arg\dfrac{z_1}{z_2} = \dfrac{\pi}{2} + 0.5404^c$

$= \boxed{2.11}$

Q2a) $f(1.6) = -1.29543..$

$f(1.7) = 0.540186..$



$\dfrac{x_1-1.6}{1.29543} = \dfrac{1.7-x_1}{0.540186}$

$\dfrac{x_1-1.6}{} \qquad \dfrac{1.7-x_1}{x_1-1.6} = \dfrac{0.540186..}{1.29543..}$

let RHS $= c$,

then $\dfrac{1.7 - x_1}{x_1 - 1.6} = c$

$1.7 - x_1 = cx_1 - 1.6c$

$x_1(c+1) = 1.7 + 1.6c$

$x_1 = \dfrac{1.7 + 1.6c}{c+1} = \boxed{1.741}$

b) $f(1.7) = -0.41612\ldots$

$f'(x) = 10x - 6x^{\frac{1}{2}}$

$f'(1.7) = 9.17696\ldots$

$x_1 = 1.7 - \dfrac{-0.41612\ldots}{9.17696\ldots}$

$= \boxed{1.745}$

Q3a) $6 \quad \left(= \frac{12}{2}\right)$

b) $y^2 = 8x$

$\dfrac{y^2}{8} = x = \dfrac{6^2}{8} = \boxed{\dfrac{9}{2}}$

c) $P\left(\frac{9}{2}, 6\right)$

$4a = 8$

$a = 2$ // $\therefore S(2, 0)$ and $P\left(\frac{9}{2}, 6\right)$

$m_{PS} = \dfrac{6-0}{\frac{9}{2} - 2} = \dfrac{12}{5}$ //

$\therefore \quad y - 6 = \frac{12}{5}\left(x - \frac{9}{2}\right)$

$y = \frac{12}{5}x - \frac{54}{5} + 6$

$\underline{\times 5}$: $5y = 12x - 54 + 30$

$5y = 12x - 24$

$12x - 5y - 24 = 0$

Q4a) $5x^2 - 4x + 1 = 0$

$\underline{\div 5}$: $x^2 - \frac{4}{5}x + \frac{1}{5} = 0$

$$\boxed{\begin{aligned} \therefore \quad \alpha\beta &= \frac{1}{5} \\ \alpha + \beta &= \frac{4}{5} \end{aligned}}$$

b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{4}{5}\right)^2 - 2\left(\frac{1}{5}\right)}{\frac{1}{5}} = \frac{\frac{6}{25}}{\frac{1}{5}} = \boxed{\frac{6}{5}}$

c) $\left(x-\left(\alpha+\frac{1}{\alpha}\right)\right)\left(x-\left(\beta+\frac{1}{\beta}\right)\right)=0$

$$\left(x^2-x\left(\alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta}\right)+\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)\right)=0$$

$$x^2-x\left(\frac{4}{5}+\frac{\beta+\alpha}{\alpha\beta}\right)+\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}=0$$

$$x^2-x\left(\frac{4}{5}+\frac{\frac{4}{5}}{\frac{1}{5}}\right)+\frac{1}{5}+\frac{6}{5}+\frac{1}{\frac{1}{5}}=0$$

$$x^2-\frac{24}{5}x+\frac{32}{5}=0$$

$\times 5$: $\boxed{5x^2-24x+32=0}$

Q5) $\underline{n=1}$: $f(1)=5+8+3=16=(4)\times 4$

$\therefore$ true for $n=1$.

assume true for $n=k$; [$f(k)=5^k+8k+3$ is div. by 4.]

consider $n=k+1$,

$f(k+1)=5^{k+1}+8(k+1)+3$

[factor out $f(k)$]

$$=5(5^k)+8k+11$$

$$=5\left[5^k+8k+3\right]-4(8k)-4$$

$$=5f(k)-4(8k+1)$$

$\therefore$ true for $n=k+1$.

so, true for $n=1$
true for $n=k+1$
when assumed true for $n=k$.

$\therefore$ By Mathematical Induction true for all $n \in \mathbb{Z}^+$

Q6a) $\displaystyle\sum_{r=1}^{n} r(r+1)(r+3) = \sum_{1}^{n} r(r^2+4r+3)$

$= \displaystyle\sum_{1}^{n} r^3 + 4r^2 + 3r = \sum_{1}^{n} r^3 + 4\sum_{1}^{n} r^2 + 3\sum_{1}^{n} r$

$= \dfrac{n^2}{4}(n+1)^2 + \dfrac{4n}{6}(n+1)(2n+1) + \dfrac{3n}{2}(n+1)$

$= \dfrac{n}{12}(n+1)\left[3n(n+1) + 8(2n+1) + 18\right]$

$= \dfrac{n}{12}(n+1)\left[3n^2 + 3n + 16n + 8 + 18\right]$

$= \dfrac{n}{12}(n+1)\left[3n^2 + 19n + 26\right]$

$= \boxed{\dfrac{n}{12}(n+1)(n+2)(3n+13)}$

b) $\displaystyle\sum_{21}^{40} \ldots = \sum_{1}^{40} \ldots - \sum_{1}^{20} \ldots$

$\displaystyle\sum_{21}^{40} r(r+1)(r+3) = \dfrac{40}{12}(41)(42)(3(40)+13)$

$- \dfrac{20}{12}(21)(22)(3(20)+13)$

$= \boxed{707210}$

Q7a) $xy = 36$

$x\frac{dy}{dx} + y = 0$ [Implicit Differentiation]

$\frac{dy}{dx} = \frac{-y}{x} = \frac{-\frac{6}{t}}{6t} = \frac{-1}{t^2}$ //

$\therefore\ y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t)$

$y = -\frac{x}{t^2} + \frac{6}{t} + \frac{6}{t}$

hence $y = -\frac{1}{t^2}x + \frac{12}{t}$

b) $\underline{(-9, 12)}$: $12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$

$12 = \frac{9}{t^2} + \frac{12}{t}$

$\underline{\times t^2}$: $12t^2 = 9 + 12t$

$12t^2 - 12t - 9 = 0$

$4t^2 - 4t - 3 = 0$

$t = \frac{3}{2},\ t = -\frac{1}{2}$ //

$t = \frac{3}{2} \rightarrow$ A $(9, 4)$

$t = -\frac{1}{2} \rightarrow$ B $(-3, -12)$

Q8a) compare with matrix for ACW rotation : $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$\left[\begin{array}{l} \theta = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \\ \theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}, \frac{2\pi}{3} \end{array}\right]$ $\left.\begin{array}{l} \cos\theta = -\frac{1}{2} \\ \sin\theta = \frac{\sqrt{3}}{2} \end{array}\right\}$ $\theta = \frac{2\pi}{3}$ fits both equations

$\therefore$ U is a $\left(\frac{2\pi}{3}\right)^c$ rotation anticlockwise about O.

b) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ so $\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ so $\begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\therefore$ $\boxed{Q = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}}$

c) $R = VU = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} & -\frac{3\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$

ii a) $S = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$

$= \begin{pmatrix} k\cos\theta & -k\sin\theta \\ k\sin\theta & k\cos\theta \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}$

$k\cos\theta = 1 \sim (1)$
$k\sin\theta = 3 \sim (2)$

$\underline{(1)^2 + (2)^2}$: $k^2\cos^2\theta + k^2\sin^2\theta = 10$

$k^2(\sin^2\theta + \cos^2\theta) = 10$

$k^2 = 10$

$\therefore k = \boxed{\sqrt{10} = \text{scale factor}}$

b) $\cos\theta = \frac{1}{\sqrt{10}}$ $\qquad$ $\sin\theta = \frac{3}{\sqrt{10}}$

$\theta = \cos^{-1}\frac{1}{\sqrt{10}} = \underline{\underline{71.6°}}$ $\qquad$ $\theta = \sin^{-1}\frac{3}{\sqrt{10}} = 1.249^c, 1.893^c$

$= \underline{\underline{71.6°}}, 108.4°$

so $\theta = 71.6°$

$\therefore$ $\boxed{\text{Anticlockwise rotation of } 71.6°}$